Get 40

Lesson Plan and Game Details for Teachers

Grade levels

"Get 40" is an adaptable game that can be played and analyzed on a variety of levels. However, we recommend that the game be used mostly in the late high school years (grades 11-12) because it does not represent a "typical" probability situation—choice is involved, and that can provoke interesting discussion among students already comfortable with the basics of probability.

Required materials

- A standard six-sided die
- Pen/pencil and paper

Instructions

- The participants draw a 2x2 grid on their paper
- The teacher rolls the die and announces the number to the group
- Players can choose to put the number into any of the four elements in the grid
- The teacher rolls again and announces again
- Players can choose to put the number into any of the remaining three elements
- This process continues until the grid is full
- Players then have the choice to add or subtract the two digit numbers (formed by rows) to try to get the target number (their "final answer")
- Goal: to get 40 as a final answer

Correlation with NCTM standards

In its most elementary characteristics, "Get 40" requires students to quickly do multiple addition and subtractions mentally in order to formulate the best move. This helps students to "understand numbers, ways of representing numbers, relationships among numbers, and number systems," as outlined in the number and operations standards for 9-12 grade education. This activity also correlates particularly well within the NCTM standards on data analysis and probability. Playing multiple rounds of "Get 40" provides students with multiple data points from which they can develop the idea of the sample space and the values of random variables, in this case the roll of a die. The NCTM standards also require that students "apply and adapt a variety of appropriate strategies to solve problems." The aspect of "Get 40" that requires students to systematically develop a strategy as the game becomes more complex fits well with this standard.

Teaching notes

The student worksheet is designed for students to learn via trial-and-error accompanied by critical thinking. The students have three chances to play the game (five rounds per chance). The first time, they play without thinking about the underlying math. Some students may begin to see some of the concepts as they play the game for the fourth or fifth time, but most probably will not.

Students then answer questions in their worksheet, leading them to a basic understanding of the game and some strategies. Teachers may wish to have students work in groups to come up with strategies. Then, groups with different strategies can compete with each other, and a debrief session can be held during which students explain the reasoning behind their strategies to the rest of the class. Alternatively, students can come up with strategies individually, and the second round can be played by the whole class together. Evaluation of strategies using specific, mathematical (or logical) reasoning is very important. Teachers should pay special attention to how students evaluate their strategies, as this is an opportunity for informal assessment. Students who really understand the strengths (or weaknesses) of a strategy will be comfortable with this question, whereas students who picked a strategy at random (or based solely on intuition) will have difficulty.

Get 40 is a great opportunity to discuss how choice affects probability of winning, and is also an example of "extreme cases" rationalization, that is, making an argument based on extreme cases. For example, students could be asked to consider the following two scenarios:

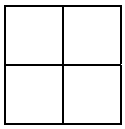
- A computer plays the game, putting the numbers randomly into any of the four boxes, and randomly chooses whether to add or subtract to get a final answer.
- An expert human plays the game with the winning strategy. If it is possible to win (i.e. if the correct numbers are rolled in the correct order) then this human wins.

The students could calculate the probability of success in each scenario (in fact, the probability of success calculated in the worksheet assumes the first scenario). The second calculation is much more involved, and is suitable only for advanced math classes in the late high school years. The teacher can also use this to discuss underlying assumptions and how they affect outcomes.

Directions

The goal of the game is to get 40. Read on to see what that means…

1. Start by drawing a 2×2 grid on a sheet of paper, like this:



2. Next, the game leader will roll a die and call out the number. Let's say the first roll is a 4. You would then place the number 4 in any one of the squares in your grid—it's your choice. Then your grid might look like this:

	4

3. Once everybody has chosen the first number's position, the game leader rolls the die again, and calls out the number. Let's say it's 6. Now you have to choose where to put the 6. Only one number can go in each spot.

4. Steps 2-3 repeat until the grid is full. Let's say it looks like this:

6	1
2	4

5. Now, combine the top two numbers to get a two-digit number (61) and the bottom two numbers to get another two-digit number (24). You can now choose to either add (61+24) or subtract (61–24) to get your final answer. The goal of the game is to make your final answer as close to 40 as possible. In this case, you would do best by subtracting 24 from 61 to get 37. You are not allowed to switch the order of the two numbers. So for example 24–61 would be an illegal move.

Play the game for five rounds and record your scores at the end of each round to see how well you do.

2×2 First Try

Round	***Final Answer***	***Difference from 40***
1		
2		
3		
4		
5		

Questions

Did you ever get exactly 40?

Using a regular (6-sided) die, how many different ways are there to get exactly 40?

How many total possible outcomes are there for this game?

Using your last two answers, during a given round, what is the probability that you are able to get exactly 40?

Now that you have played the game a few times, you are probably beginning to think of some strategies. Write down some strategies and use them playing five more rounds. (Hint: It may help to think about your strategy for each move you make in a round, in other words, the first move may require a different strategy than the second move).

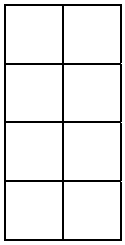
List of Strategies:

2×2 Second Try

Round	***Final Answer***	***Difference from 40***
1		
2		
3		
4		
5		

Did you do any better using specific strategies than in the first round? Why did your successful strategy work (or your unsuccessful one fail)?

Finally, let's add a new twist by doubling the size of the board to be four rows by two columns. You now have eight digits to place and three decisions to make about addition and subtraction.



Are there any ways in which your strategy should change for this version?

4×2 Board Score

Round	***Final Answer***	***Difference from 40***
1		
2		
3		
4		
5		

Does the larger board make it harder or easier to get close to 40? Explain your reasoning.

Game Details

-Required materials: one standard die (numbered 1-6) per group of players, pen/pencil, paper
-Procedure:

- The participants draw a 2x2 grid on their paper
- The teacher rolls the die and announces the number to the group
- Players can choose to put the number into any of the four elements in the grid
- The teacher rolls again and announces again
- Players can choose to put the number into any of the remaining three elements
- This process continues until the grid is full
- Players then have the choice to add or subtract the two digit numbers (formed by rows) to try to get the target number

-Goal: get to the target number (which in this case is 40)

Play the game for five rounds and record your scores at the end of each round to see how well you do.

Questions

Did you ever get exactly 40?

The answers will vary, but it is not particularly likely that students will get 40 for a particular round.

Using a regular (6-sided) die, how many different ways are there to get exactly 40?

The easiest way to solve this is by case. This can be done by trial-and-error or by a somewhat more systematic and rigorous method, as follows:

- Note that, for the final answer to be exactly 40, the top and bottom tens digits can only be certain combinations of numbers as follows:

6	a
2	b

5	c
1	d

1	e
2	f

2	g
1	h

- In cases 1 and 2, subtraction is required to get to 40, and thus a = b = 1,2,3,4,5,6 and c = d = 1,2,3,4,5,6. (Yields 12 possible boards.)
- In cases 3 and 4, since the tens digit currently add to 3, the ones digits must sum to 10 to get 40. This requires that e = 6 and f = 4 or e = f = 5 or e = 4 and f = 6. The same thing holds for g and h. (Yields 6 possible boards.)
- So the total number of ways to get 40 is 12 + 6 = **18**.

How many total possible outcomes are there for this game?

Since each roll is independent, and the question asks for **total** outcomes, the answer is $2 \times 6^4 =$ **2592**. The factor of 2 is due to the fact that you can either add or subtract.

Using your last two answers, during a given round, what is the probability that you are able to get

exactly 40?

18 / 2592 = 0.694 %
The students should realize that this is extremely small. One way to have them think about this is to ask them if they would bet money on winning (say, if this game were in a casino). More advanced students can think about the ways in which the probability could be increased or reduced to suit the gambler or the house, respectively. Roulette is an example of a casino game with a small probability of success for the gambler (though not as small as our game).

Now that you have played the game a few times, you are probably beginning to think of some strategies. Write down some strategies and use them playing five more rounds. (Hint: It may help to think about your strategy for each move you make in a round, in other words, the first move may require a different strategy than the second move).

List of Strategies:

The students should devise strategies based on the 18 winning combinations. They should realize that there is more flexibility in the ones column than in the tens, and so, if a 1, 2, 5, or 6 comes up on the first turn, they should place it in the tens column. Furthermore, every winning combination has a 1 or a 2 in the bottom-left quadrant, so if either of those numbers is rolled on the first turn, it should be placed bottom-left. Likewise, none of the winning combinations have a 3 or 4 in the tens, so they should always be placed in the ones column.

More advanced classes can draw tree diagrams, based on their chosen strategies, for the placement of four digits to reach the 18 winning combinations, and can therefore calculate the probability of success when using any given strategy.

2×2 Second Try

Round	***Final Answer***	***Difference from 40***
1		
2		
3		
4		
5		

Did you do any better using specific strategies than in the first round? Why did your successful strategy work (or your unsuccessful one fail)?

Students should understand that it is perfectly OK to guess at an incorrect strategy and then discover that it was wrong—as long as they now understand why that strategy didn't work.

Finally, let's add a new twist by doubling the size of the board to be four rows by two columns. You now have eight digits to place and three decisions to make about addition and subtraction.

Are there any ways in which your strategy should change for this version?

Students should recognize that changing the parameters of the game will affect the outcome. Their strategy may or may not vary from the 2x2 board, but students should see that the element of more choice (i.e. With three additions/subtractions instead of one), the optimal strategy may change. The students should also realize that they can get much closer to 40 by carefully playing the 4x2 board, as they have more control over their final score than the high randomness of the 2x2 version.

4×2 Board Score

Round	***Final Answer***	***Difference from 40***
1		
2		
3		
4		
5		

Does the larger board make it harder or easier to get close to 40? Explain your reasoning.

Students should be perplexed about the meanings of "harder" and "easier." They should struggle with the translation of these terms into mathematical parlance. There are two answers, each based on its own (equally correct) interpretation of the meaning of the above terms:

1. Harder, because the total number of boards is now $3 \times 6^8 = 5{,}038{,}848$ and the number of winning combinations is expected to increase approximately linearly (either mathematical reasoning or intuition is fine here).
2. Easier, because the player can make more choices and therefore "fine tune" his/her answer to better get the target number.

The first answer is based on a numerical definition of "easier" based solely on probability. By this interpretation, a harder game would be one with a lower probability of success.

The second answer is based on the "ease of use" perspective. In other words, because the player can now make many more decisions (about where to place the numbers and what to add or subtract) he/she can better prepare for the uncertainty involved in the rolling of the die. One way to measure this is with the concept of "game control," i.e. what percentage of moves cannot be decided by the player. In the 2×2 case, this percentage is $1 / 5 = 20\%$. In the 4×2 case, this percentage is $1 / 11 = 9\%$.

Whatever the answer, the students should justify their reasoning.