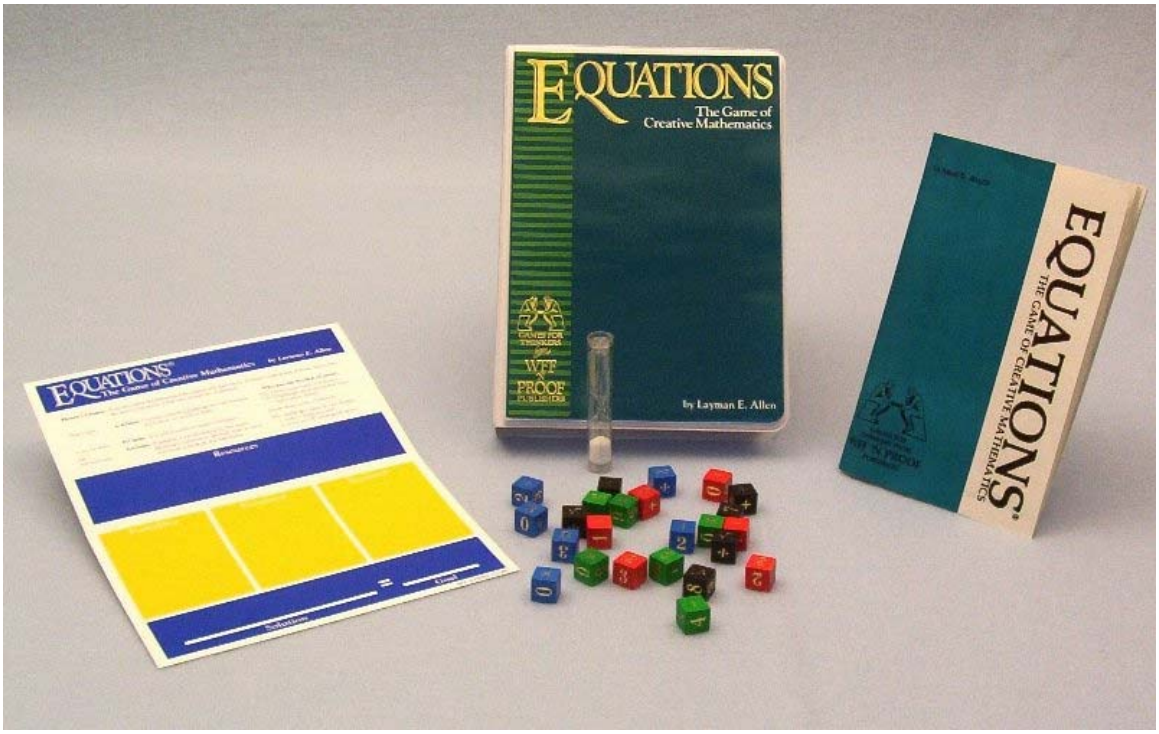
EQUATIONS - STUDENT HANDOUT



Game Background:

The game consists of a playing mat and 24 cubes. Each cube contains four digits and two operation signs (+, -, *, ×, ÷, √). For each round, up to six cubes may be used to create a Goal. The players then must form a Solution equal to the Goal using the remaining cubes (the Resources).

Key:

+	addition
-	subtraction
*	exponent
×	multiplication
÷	division
√	root

Student Handout - Continued

Directions:

To begin, each player rolls a red cube. The player with the highest numeral gets to set the Goal. That player rolls all 24 cubes, choosing up to six of them to make the Goal. The remaining cubes are placed into the Resources section of the game board.

After the Goal has been set, play progresses to the left. When it is your turn to play, you must either give a Solution, or move a cube from Resources into the Forbidden, Permitted, or Required section.

Cubes placed into the Forbidden section must not be used in the Solution. Cubes placed in the Required section must be used in the Solution. Cubes placed in the Permitted section may or may not be used in the Solution.

Play proceeds with each player moving a block from the Resources section into one of the other sections on every turn.

In order to give a Solution, a player must propose it before moving a block. The player then uses one block from the Resources section, all blocks from the Required section, and any necessary blocks from the Permitted section in order to reach the Goal. The first player to give a valid solution wins the round.

Each turn must take place before the sand timer runs out. If time runs out, the player forfeits his turn.

Game variations:

1. <u>Order of operations:</u>

Order of operations may be used in multiple different ways. Before each round, the group must agree on which way order of operations will be used. Standard order of operations may be enforced. So, $3 + 2 \times 2$ will always equal $3 + (2\times2) = 7$.

OR, students may agree to permit the use of parentheses in the Goal and the Solutions. So, $3 + 2 \times 2$ could be treated as $(3 + 2) \times 2 = 10$. For example, the blocks 3, 4, √, ×, 2, + may be used in the following ways:

$\sqrt[3]{(4\times2)}$ = third root of 8 = 2
$\sqrt{(4)} + (3\times2) = 8$
$(\sqrt{(4)} + 3) \times 2 = 10$

2. Sideways rule:

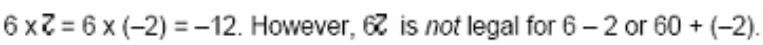
Student Handout - Continued

The sideways rule will help students become comfortable using fractions. When using the sideways rule, a number block may be turned sideways in the Goal or Solution to equal the reciprocal of that number. For example:

$1 \div \text{[sideways 3]} = 1 \div (1/3) = 1 \times 3 = 3$

3. Upside-down rule:

The upside-down rule will help students become comfortable with negative numbers. When using the sideways rule, a number block may be turned upside-down in the Goal or Solution to equal the additive inverse of that number. For example:



$6 \times \text{[upside-down 2]} = 6 \times (-2) = -12$. However, [upside-down 2] is *not* legal for $6 - 2$ or $60 + (-2)$.

Any of these rules may be implemented at any time to make the game more challenging for older students.

Questions:

As you play, consider strategies that will help you to get numbers close to the Goal:

- What are the best operations to use when the goal is a small number?
- What are the best operations to use when goal is a large number?
- How can the sideways rule be used to make very large numbers?
- How can the sideways rule be used to make very small numbers?
- What are strategic ways to use the numbers 1 and 0?

After you play, consider the following questions:

- What are the best strategies? What should the players be considering when deciding whether to move a block into Required, Forbidden, or Permitted?
- Is it easier to reach the Goal when the Goal is small or large?
- Can you think of any other special rules that might make the game more interesting?

Teacher Lesson Plan

Title: Equations

Game Type: Use mathematical notation, develop skills, devise strategies.

Materials: Equations blocks, game board, sand timer.

(The materials are available at http://wffnproof.com/store/math-logic/product_0002.html)

Learning Objectives:

- The student will be able to use order of operations, exponents, and fractions to creatively solve problems.
- The student will be able to solve the same problem in multiple different ways, using different operations.
- The student will develop an overall fluency and comfort level with arithmetic.

Target Audience:

Middle school through high school students taking algebra I.

Where it Fits in the Curriculum:

All students should be familiar with exponents and square roots. More advanced play in the game may take place if students are familiar with higher roots (i.e., cube roots), reciprocals, operations with fractions, operations with negative numbers, and order of operations.

Reference to NCTM Learning Standards (Number and Operations Standards)

- Work flexibly with fractions, decimals, and percents to solve problems;
- Understand the meaning and effects of arithmetic operations with fractions, decimals, and integers;
- Use the associative and commutative properties of addition and multiplication and the distributive property of multiplication over addition to simplify computations with integers, fractions, and decimals;
- Understand and use the inverse relationships of addition and subtraction, multiplication and division, and squaring and finding square roots to simplify computations and solve problems.
- Judge the effects of such operations as multiplication, division, and computing powers and roots on the magnitudes of quantities;
- Select appropriate methods and tools for computing with fractions and decimals from among mental computation, estimation, calculators or computers, and paper and pencil, depending on the situation, and apply the selected methods;

Teaching Notes:
Play a sample game with the whole class first to demonstrate the rules and what a player can do on his or her turn. You may want to copy the game board on transparency and project it on an overhead projector. Since the dice are solid and cannot be projected, you may want to keep the dice on a separate game board, and write the numbers shown on the projected game board, changing them as the dice are moved around. If erasing and rewriting is a chore, you can also cut up several sets of small transparency pieces with the printed digits from 1-6 to manipulate on the projected game board.

Gameplay Analysis:

Equations creates the opportunity for players to become extremely familiar with every different type of operation, and how to manipulate numbers and operations to reach a desired goal. There are so many different possible strategies that each student will develop his own style of play. Different styles will create differing levels of difficulty:

Students who tend to move blocks into the Required section of the game board will make it more difficult to reach the goal. These students will need to be creative in using all of the Required blocks. Students tend to reach their own level when playing the game. That is, they make the game as difficult as they want it to be. This can be very beneficial in a heterogeneous classroom and can allow students of varying abilities to all play together.

Answers to Questions on Student Handout:

- What are the best operations to use when the goal is a small number?
    - Generally, addition and subtraction allow the player to stay close to a given number. A square root or division can also significantly reduce a number's size.
- What are the best operations to use when goal is a large number?
    - Multiplication can make the numbers very large very fast. The exponential will make the largest numbers.
- How can the sideways rule be used to make very large numbers?
    - The sideways rule creates fractions. Dividing by fractions is the same as multiplying, which can create large numbers.
- How can the sideways rule be used to make very small numbers?
    - Multiplying by fractions is the same as division, which makes numbers much smaller.
- What are strategic ways to use the numbers 1 and 0?

    - The numbers 1 and 0 can be used to get of pesky Required blocks without getting too far away from the current number. Multiplying (or dividing) by 1 or adding (or subtracting) 0 will not change the value.

- What are the best strategies? What should the players be considering when deciding whether to move a block into Required, Forbidden, or Permitted?

    - This question is very open-ended. Moving blocks into Required generally makes the Goal more difficult to reach, while moving blocks into Permitted makes the Goal easier to reach. More advanced students will probably find it advantageous to move blocks into Required, while less advanced students will probably move blocks into Permitted or Forbidden. Asking this question to the class for discussion will help generalize strategies and allow for multiple different successful strategies to surface.

- Is it easier to reach the Goal when the Goal is small or large?

    - In general, the goal is easier to reach when it is smaller.

- Can you think of any other special rules that might make the game more interesting?

    - One example might be a wild rule, in which the number 0 (or some other number/operation) can represent any number. There are many possible special rules, which can make the game more challenging and more fun.

Example game board answers:

The example game boards in the attached pdf entitled EXAMPLES should be given to students after playing the game for a while. They will demonstrate to the teacher if the students caught on to some of the items hinted at with the questions, such as the importance of 1 and 0, and the ability to manipulate order of operations. You can also ask the students to look for ways to use the sideways and upside-down rules to get solutions.

Following are possible solutions (remember, all solutions must use only one block from Resources!):

1. 3×3
   9×1

2. $(6+6) \times \sqrt{(3\div3)}$
   $\sqrt{(9)} + (3\times3) \div 1$
   $\sqrt{(3\times3)} + 9 \div 1$
   There are many more solutions to this example!

3. $(9*3) + (0 \times (7 \div (\sqrt{(9)} - 8)))$
   There are many more solutions to this example!

Note the use of zeros and ones in the previous examples. They can be incredibly useful if a student is able to reach the goal but has not yet used all of the required blocks.

Equations Game Board

Resources:		
FORBIDDEN	PERMITTED	REQUIRED
	GOAL: _____	
Solution:		